Dynamic Multipath Estimation by Sequential Monte Carlo Methods

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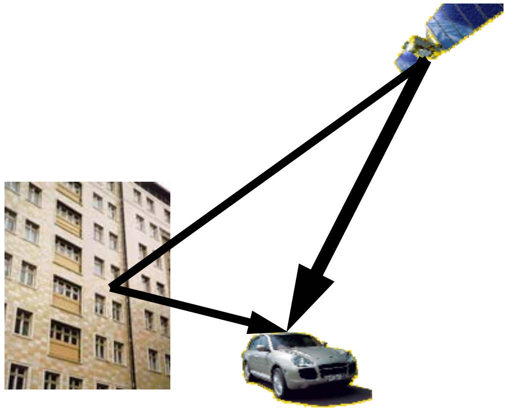
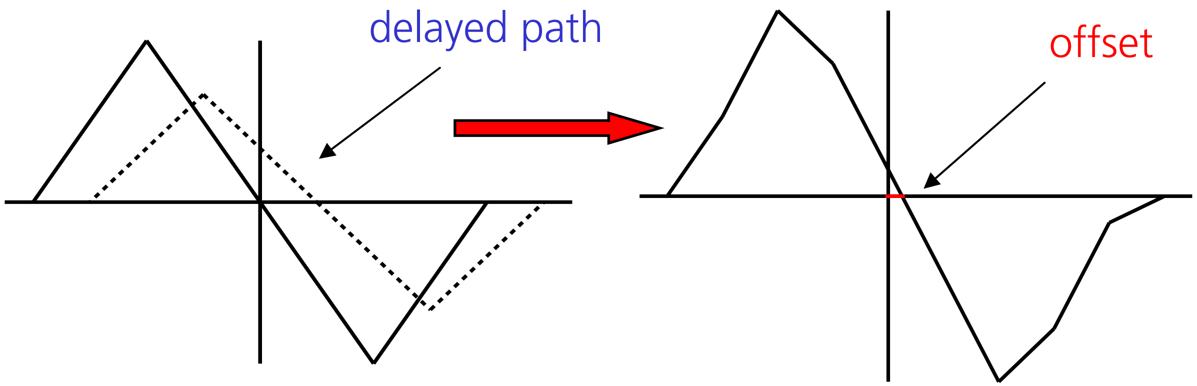
Outline

- **Multipath problem and signal model**
- **Maximum-likelihood delay estimation**
  - Efficient likelihood computation
  - Limitations of ML estimation
- **Sequential Bayesian estimation**
  - Incorporation of channel characteristics
  - Particle Filter implementation
- **Simulation results**
  - Comparison with conventional DLL (narrow correlator)

The Effect of Multipath

- **Multipath:** superposition of received signals with different amplitude and delay





- Offset in Loop-S curve: DLL estimate gets biased → error due to multipath

- Signal with multipath at the receiver:

$$z(t) = \sum_{i=1}^{N_m} a_i(t) \cdot [c(t) * g(t - \tau_i(t))] + n(t)$$

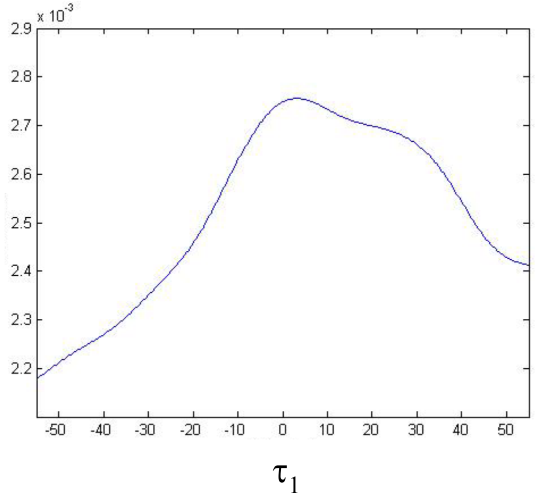
Maximum Likelihood Estimation

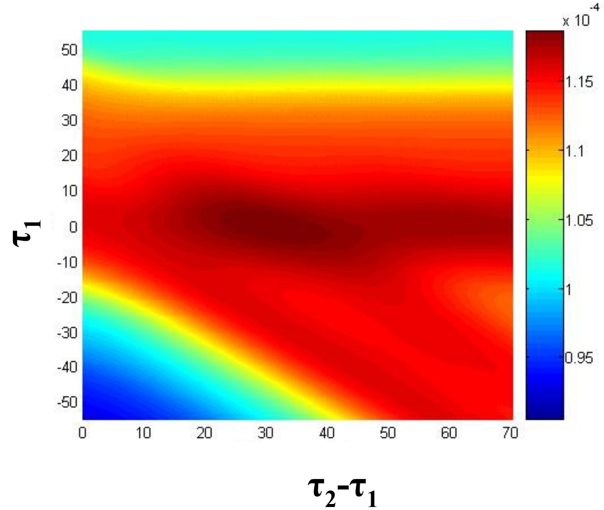
- **Discrete Signal Model:**

$$z(t) = \sum_{i=1}^{N_m} a_i(t) \cdot [c(t) * g(t - \tau_i(t))] + n(t) \quad \Longrightarrow \quad \mathbf{z}_k = \underbrace{\mathbf{C}\mathbf{G}(\boldsymbol{\tau}_k)\mathbf{a}_k}_{\mathbf{s}_k} + \mathbf{n}_k$$

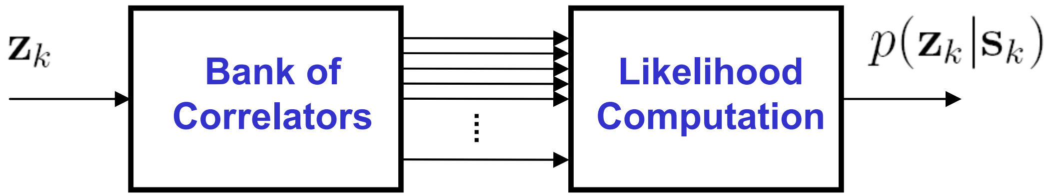
- **Likelihood function:**

$$p(\mathbf{z}_k|\mathbf{s}_k) = \frac{1}{(2\pi)^L \sigma^{2L}} \cdot \exp\left[-\frac{1}{2\sigma^2}(\mathbf{z}_k - \mathbf{s}_k)^H (\mathbf{z}_k - \mathbf{s}_k)\right]$$





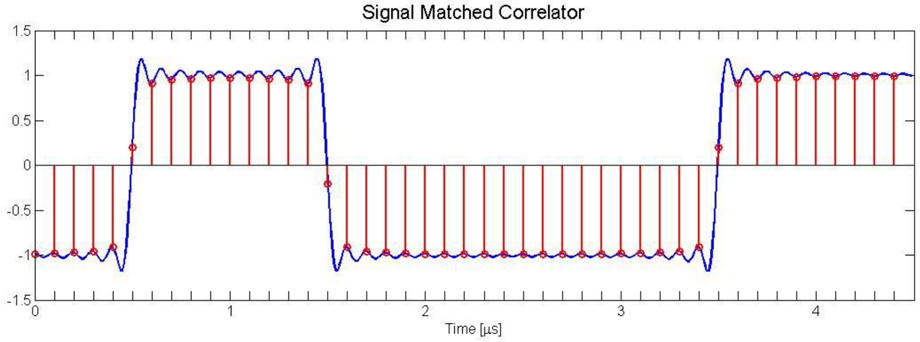
Efficient Likelihood Computation



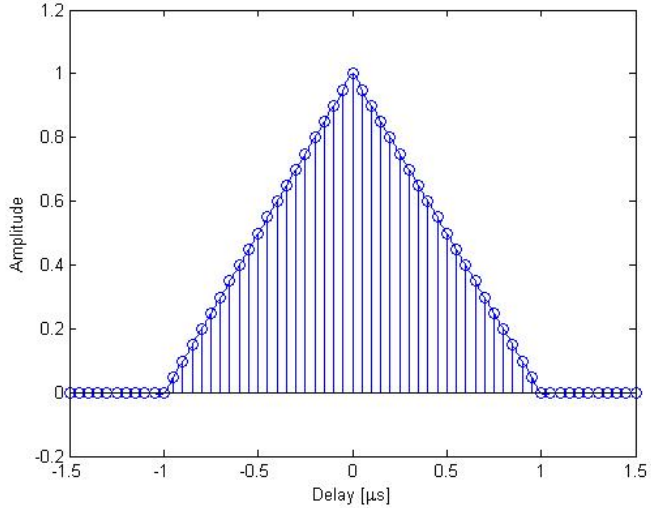
Data size reduction:
- Estimation within subspace
- Correlator outputs are sufficient statistics for delay estimation

Fourier Interpolation:
- Continuous time-shifts possible
- Independent of sampling rate

Signal-matched correlators:



Correlator Outputs:



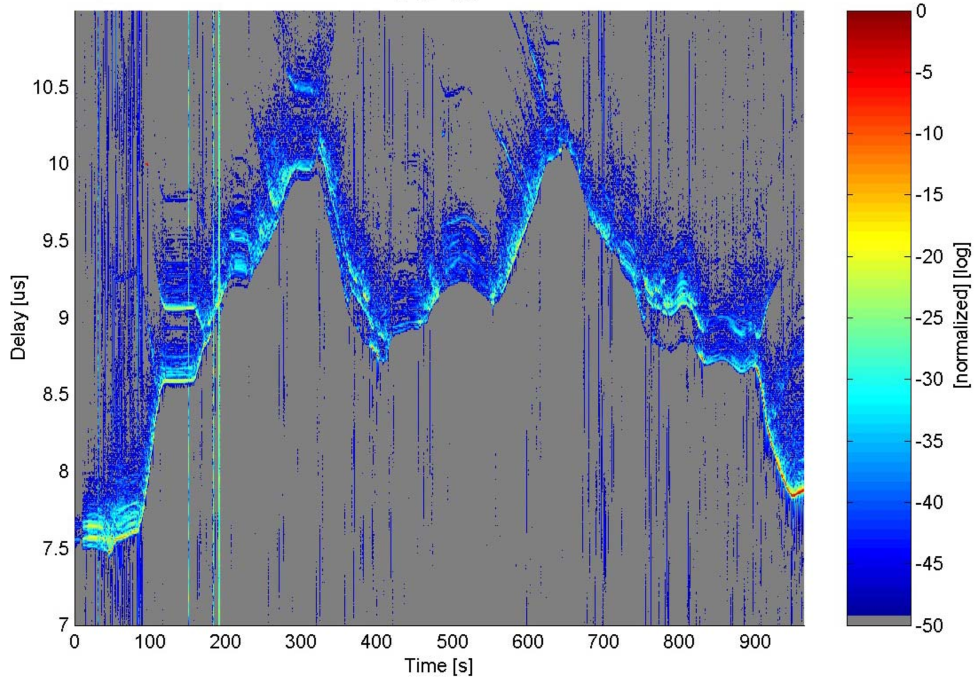
Limitations of ML Estimation

Measurement vector:

$$\mathbf{z}_k = \underbrace{\mathbf{C}\mathbf{G}(\boldsymbol{\tau}_k)\mathbf{a}_k}_{\mathbf{s}_k} + \mathbf{n}_k$$

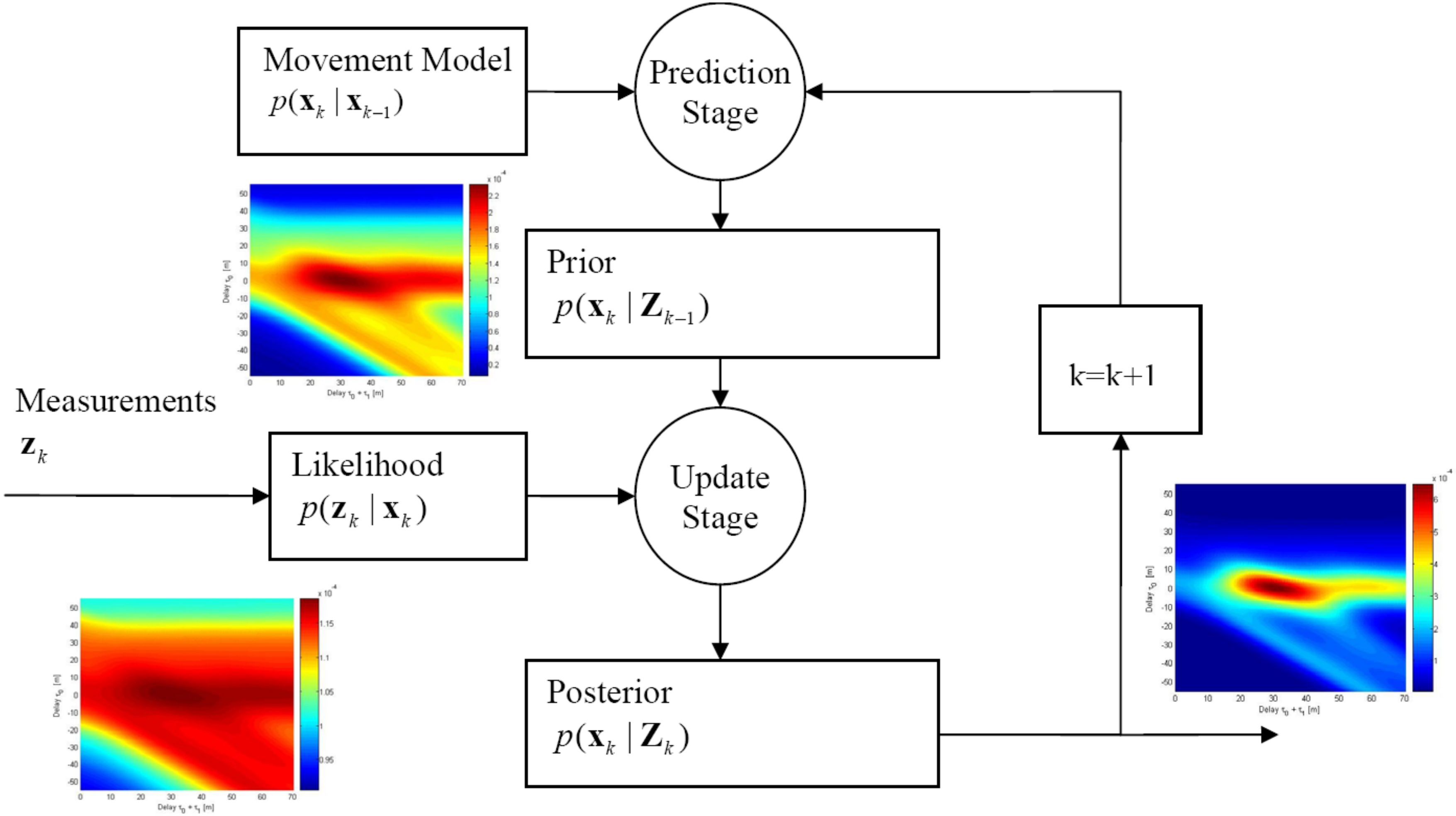
Parameters assumed constant during observation time

Urban channel measurement:



- **ML estimator requires adjustments:**
  - **Short coherent integration time → measurements very noisy**
  - **Longer integration time → parameter changes violate assumptions**
- **Dependence of consecutive time instances not modeled adequately**

Sequential Bayesian Estimation

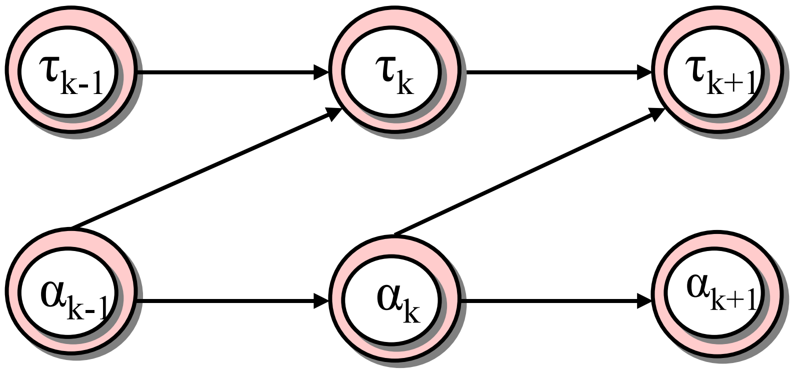


Movement Model: Characterization of the Channel

- **State vector:**

$$\mathbf{x}_k = \left[\tau_{1,k}, \Delta\tau_{2,k}, \ldots, \Delta\tau_{N_m,k}, \alpha_{1,k}, \ldots, \alpha_{N_m,k}\right]^T$$

- **State transition process:** $p(\mathbf{x}_k | \mathbf{x}_{k-1})$ **(Markov model)**



$$\tau_{1,k} = \tau_{1,k-1} + \alpha_{1,k-1} \cdot \Delta t + n_\tau \quad ,$$

$$\Delta\tau_{i,k} = \Delta\tau_{i,k-1} + \alpha_{i,k-1} \cdot \Delta t + n_\tau$$

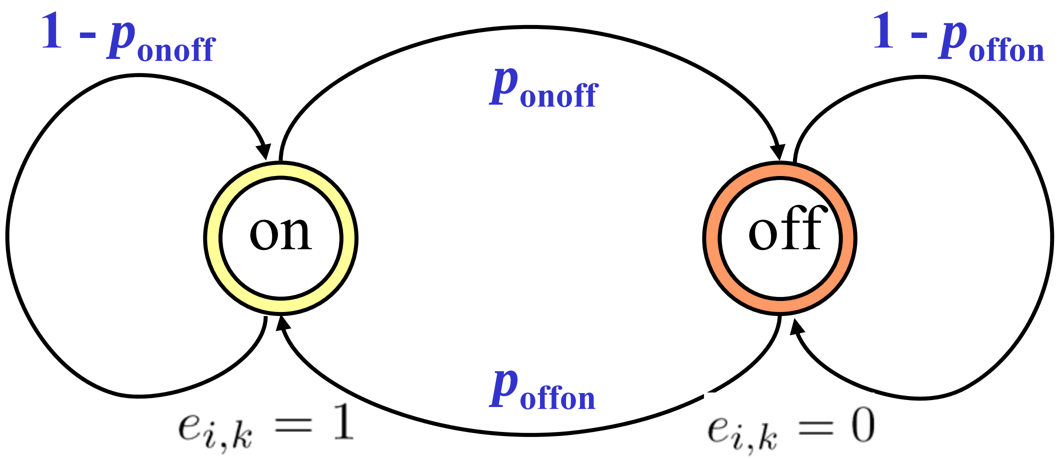
$$\alpha_{i,k} = \left(1 - \frac{1}{K}\right) \cdot \alpha_{i,k-1} + n_\alpha$$

Movement Model with Path Activity Tracking

- **State vector:**

$$\mathbf{x}_k = \left[\tau_{1,k}, \Delta\tau_{2,k}, \ldots, \Delta\tau_{N_m,k}, \alpha_{1,k}, \ldots, \alpha_{N_m,k}, \underbrace{e_{1,k}, \ldots, e_{N_m,k}}_{\text{path activity}}\right]^T$$

- **Markovian multipath activity model:**

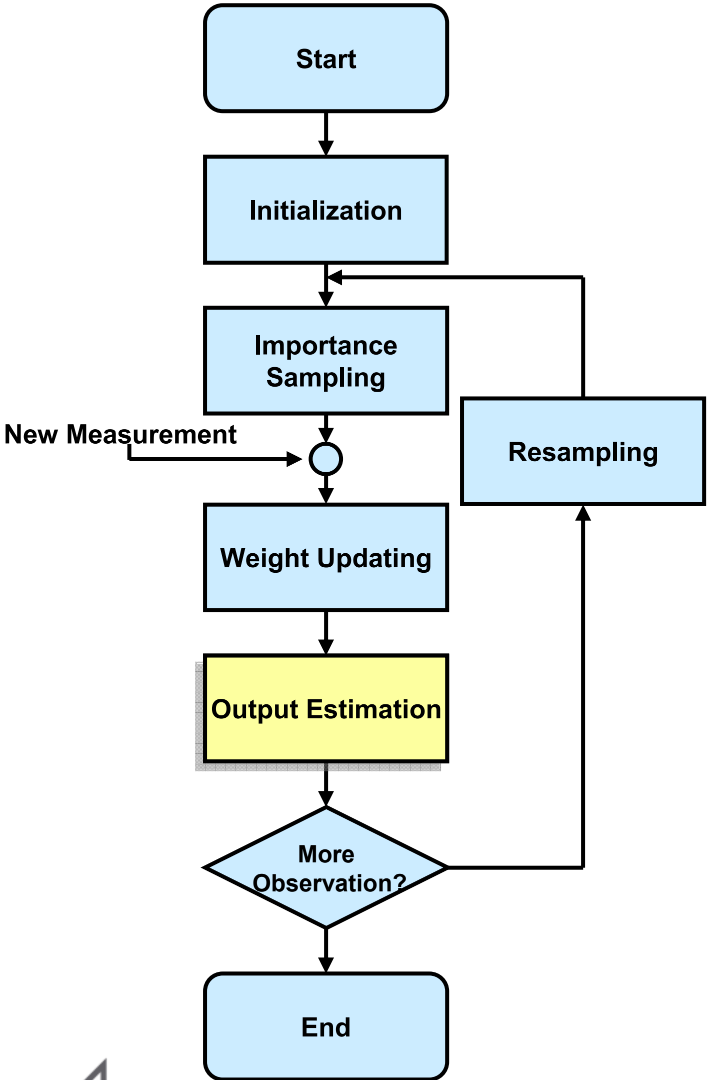


$$e_{i,k} \in \{1 \equiv \text{"on"}, 0 \equiv \text{"off"}\}$$

$$p(e_{i,k} = 0 | e_{i,k-1} = 1) = p_{\text{onoff}}$$

$$p(e_{i,k} = 1 | e_{i,k-1} = 0) = p_{\text{offon}}$$

Sampling Importance Resampling Particle Filter

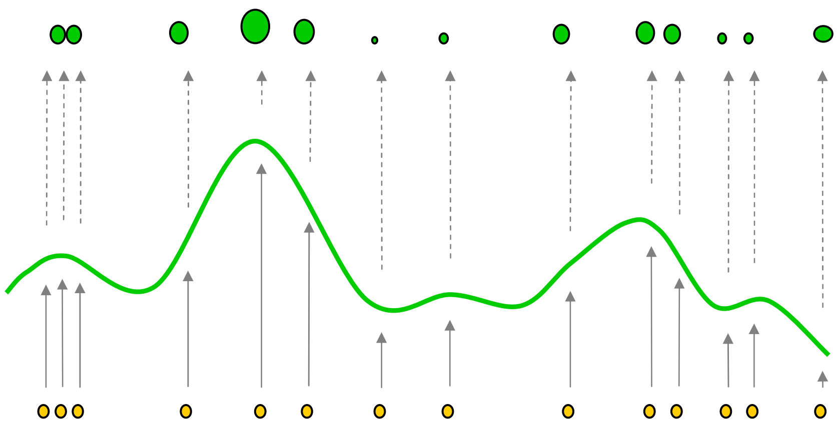


Maximum A Posteriori

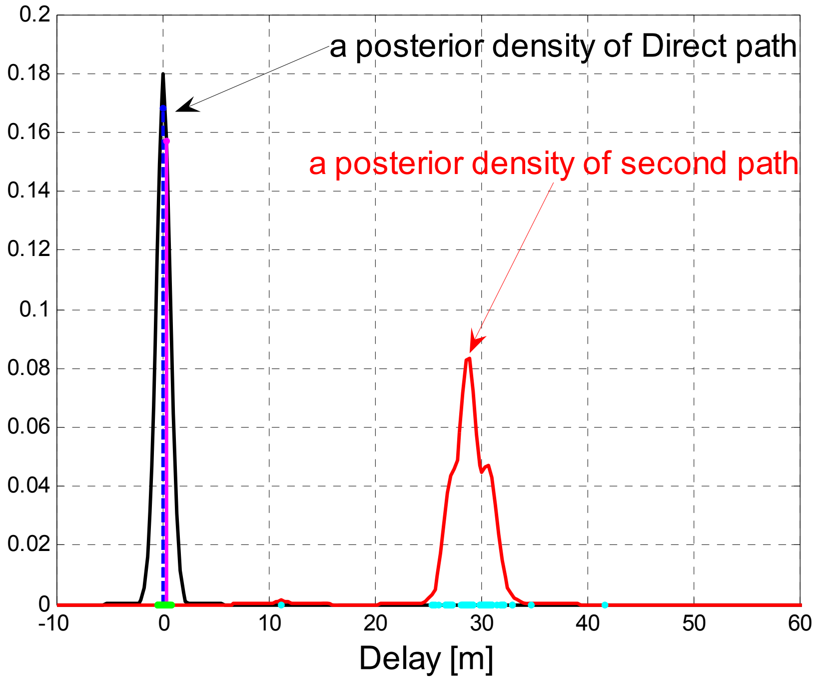
$$\hat{x}_k^{MAP} = \arg\max_{x_k} p(x_k \mid Z_k)$$

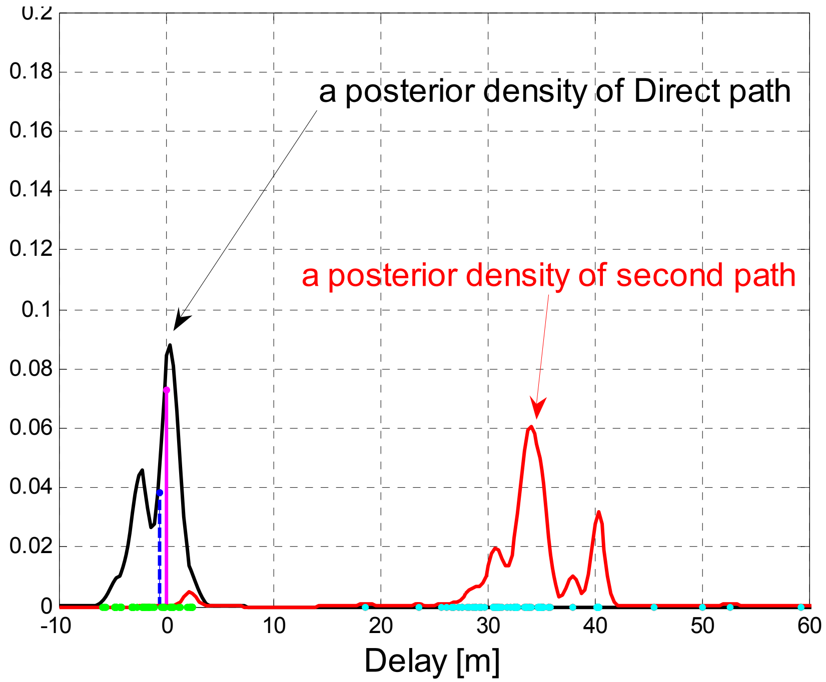
Minimum Mean Square Error

$$\hat{x}_k^{MMSE} = \int x_k\, p(x_k \mid Z_k)\, dx_k$$



Posterior Distribution

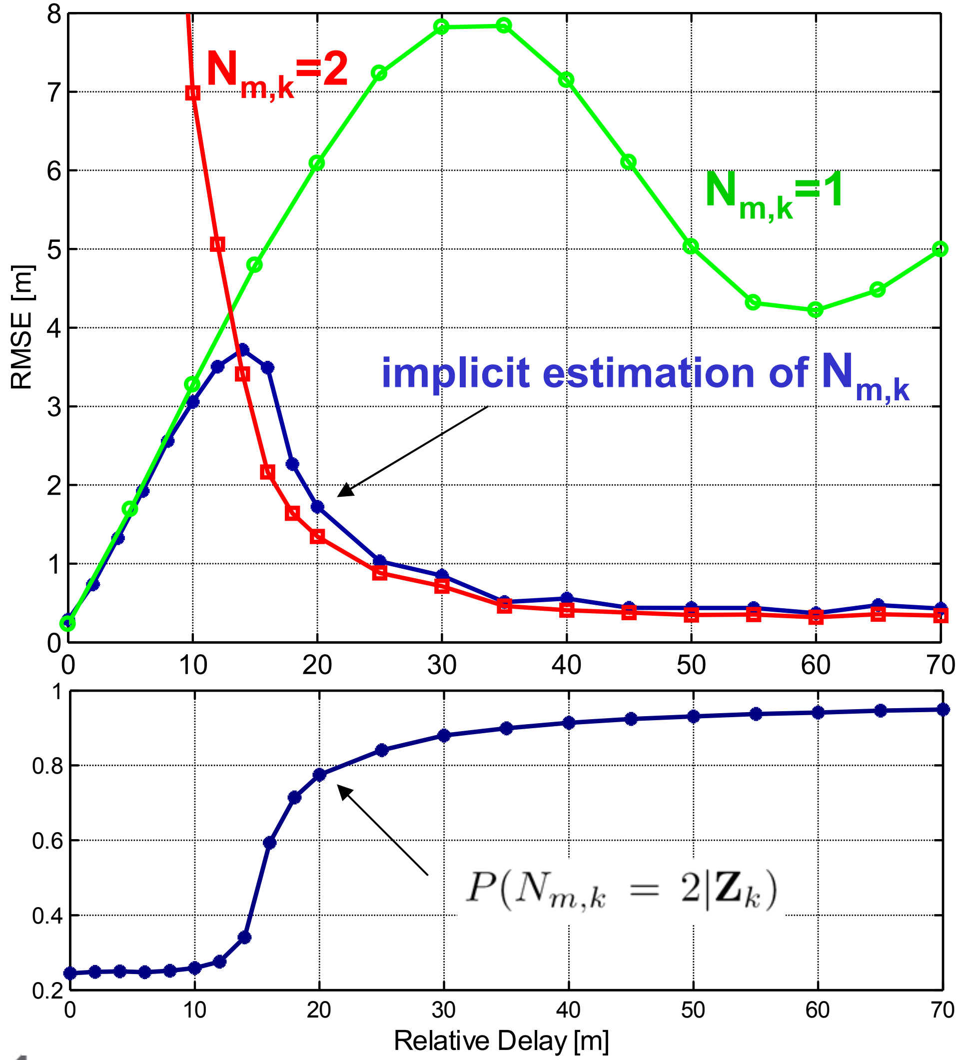




- **Represented by particles:** $$p(\mathbf{x}_k|\mathbf{Z}_k) \approx \sum_{j=1}^{N_p} w_k^j \cdot \delta(\mathbf{x}_k - \mathbf{x}_k^j)$$

- **Contains all uncertainty about range:**
  perfect for sensor data fusion (also non-Gaussian and multi modal PDFs)

Simulation Results: Static Multipath



Channel example:

Static multipath

Relative amplitude: 0.5

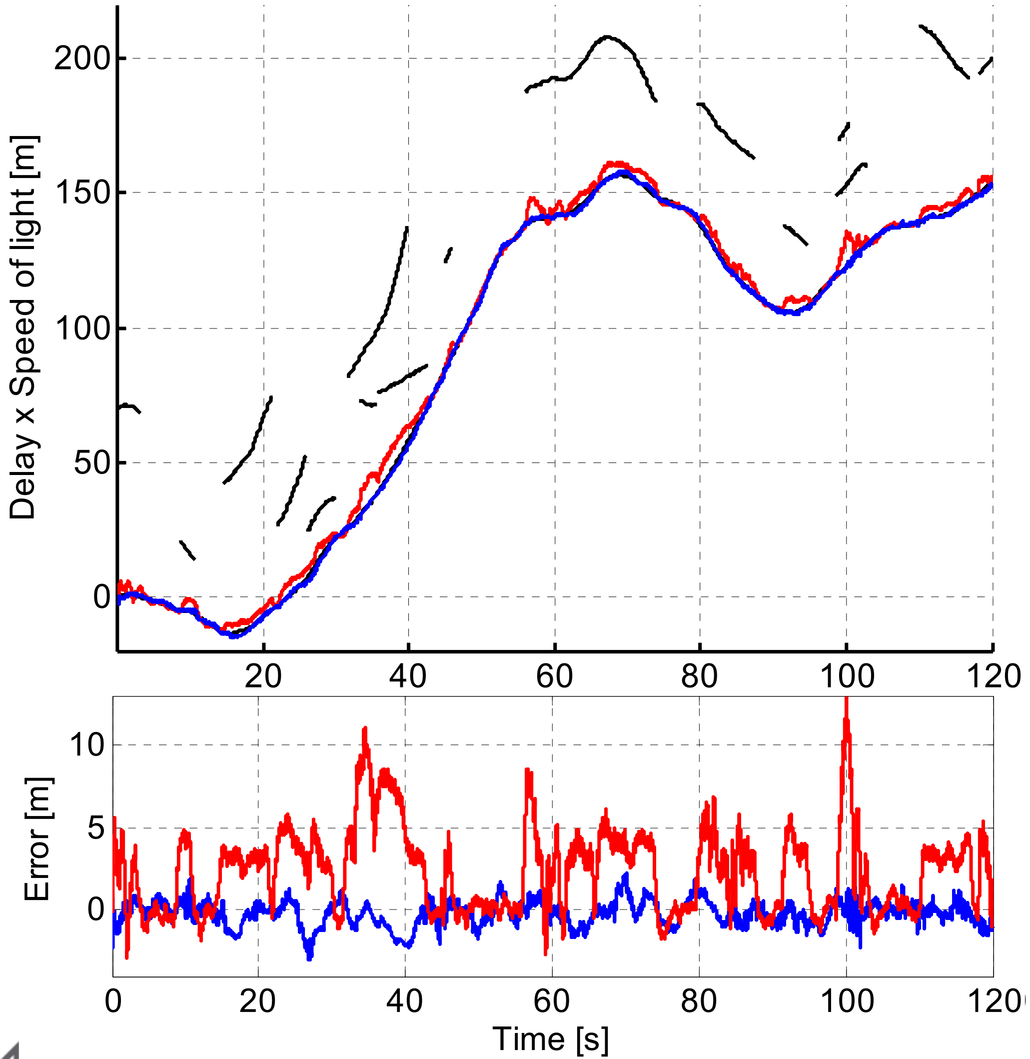
C/N_0: 45 dB-Hz

GPS C/A signal

The model implicitly represents number of paths:

$$N_{m,k} \hat{=} \sum_{i=1}^{N_m} e_{i,k}$$

Simulation Results: Dynamic Multipath



Channel example:

Up to N_m=3 paths

Relative amplitude: 0.5

C/N_0: 45 dB-Hz

GPS C/A signal

DLL with 0.1 chip spacing

RMSE: 3.49 m

Particle Filter with N_s=20000

RMSE: 0.77 m

Directions of Future Work

- **Improved movement models**
  - Adaption to measured channels
  - Complexity/performance tradeoffs
- **Bayesian estimator in the position domain**
  - Complexity/performance tradeoffs
- **Utilization of posterior PDFs**
  - Integrity
  - Carrier phase positioning
  - Combination with other “sensors”