A SINGLE-STAGE TARGET TRACKING ALGORITHM FOR MULTISTATIC DVB-T PASSIVE RADAR SYSTEMS

Xuefeng Yin\textsuperscript{1}, Troels Pedersen\textsuperscript{2}, Peter Blattning\textsuperscript{3}, Alain Jaquier\textsuperscript{3}, and Bernard H. Fleury\textsuperscript{2,4}

\textsuperscript{1}College of Electronics and Information Engineering, Tongji University, Shanghai, China
\textsuperscript{2}Section Navigation and Communications, Department of Electronic Systems, Aalborg University, Denmark
\textsuperscript{3}Federal Department for Defence, Civil Protection and Sports, armasuisse, Science and Technology, Switzerland
\textsuperscript{4}Telecommunications Research Center Vienna (fhw.), Vienna, Austria

Email: xuefyin@gmail.com

ABSTRACT
In this contribution, a particle filter (PF) is proposed for tracking a single target illuminated with a digital-video-broadcasting terrestrial (DVB-T) signal in a multistatic radar system. This algorithm utilizes a single-stage scheme, i.e. it estimates the target position trajectory in a Cartesian coordinate system directly from the signal reflected by this target. The multiple stages adopted in the conventional approaches are unnecessary. The proposed algorithm augmented with successive interference cancellation can be used to track individual targets in a multiple-target scenario. Simulation studies show that this PF exhibits lower complexity and better performance than two conventional algorithms using multiple stages.

Index Terms— Target tracking, multistatic passive radar, maximum likelihood estimation, terrestrial digital-video-broadcasting, and particle filtering.

1. INTRODUCTION

Target tracking algorithms available in open literature all operate in successive multiple stages. For instance, the methods proposed in \cite{1, 2} have three successive stages, which are \textit{i}) estimation of the parameters, such as delay (time of arrival), Doppler frequency and angle of arrival, of the signals reflected by targets, and the detection of the target signals using thresholding techniques \cite{2}, \textit{ii}) association of the parameter estimates to individual tracks, one for each target, and \textit{iii}) estimation of the target (position) trajectories in a Cartesian coordinate system from these tracks. The latter two stages are combined as one stage in the particle-filter (PF)-based algorithms described in \cite{3, 4}. The recently proposed track-before-detection (TBD) algorithms \cite{5, 6} also utilize a two-stage scheme. In the first stage, the dispersion power spectrum, e.g. in delay and Doppler frequency, of the received signal is estimated. Then the target trajectories are extracted in the second stage. Following the nomenclature used in \cite{7}, we refer to the (complex baseband) signal reflected by a target at the output of the radar receiver as a “target signal” and the parameters characterizing this signal as the “target signal parameters”.

The multi-stage-based tracking algorithms have two disadvantages: \textit{i}) the estimates of the target trajectories are not maximum likelihood (ML) estimates, even in the case where ML likelihood methods are used to estimate target signal parameters, and extract target trajectories sequentially \cite{8}; \textit{ii}) these tracking algorithms calculate the estimates of target signal parameters or the dispersion power spectrum \textit{independently} from individual observations, i.e. without exploiting the dynamic behavior of the targets. This procedure leads to a loss of information and consequently to a performance degradation.

In this contribution, we propose a PF-based single-stage tracking algorithm which estimates the target trajectory in the Cartesian coordinate system directly from the target signal without performing the intermediate stages, such as target signal parameter estimation and track association. To our best knowledge, this PF is the first single-stage algorithm for target localization and tracking using electromagnetic waves. We use this algorithm to track targets illuminated with digital-video-broadcasting terrestrial (DVB-T) signal in a multistatic passive radar system. The performance of this PF is compared with those of two conventional algorithms by means of Monte-Carlo simulations.

The organization of the paper is as follows. Section 2 presents the state-space model of the target movement and the observation model of the target signal. In Section 3, the
The proposed PF algorithm is formulated. The performance of the algorithm is investigated by means of simulations in Section 4. Concluding remarks are addressed in Section 5.

2. SIGNAL MODEL

In this section, the movement of a target in a 2-dimensional Cartesian coordinate system is described by a state-space model. The observation model of the target signal is presented. We consider a single-target scenario for the presentation.

2.1. The State-Space Model

Tracking the trajectory of a target can be viewed as the estimation of the state of a dynamical system from observations. In our case, the dynamic behavior of the state is described by the linear model

\[ \theta_k = F_k \theta_{k-1} + G_k n_{k-1}, \quad k = 1, \ldots, K, \]

where the state vector \( \theta_k = [p_k^T \quad v_k^T]^T \) with \((\cdot)^T\) representing the transpose operation consists of the position \( p_k \in \mathbb{R}^2 \) and the velocity \( v_k \in \mathbb{R}^2 \) of a target at the beginning of the \( k \)th observation period in a 2-dimensional Cartesian coordinate system, the driving process \( n_k \in \mathbb{R}^2 \), which is assumed constant within the \( k \)th observation period, accounts for the acceleration due to the unknown "manoeuvre" input, \( K \) is the total number of observation periods, and the matrices \( F_k \in \mathbb{R}^{2 \times 4} \) and \( G_k \in \mathbb{R}^{2 \times 2} \) are defined as respectively

\[ F_k = \begin{bmatrix} I_2 & T \cdot I_2 \\ 0_2 & I_2 \end{bmatrix} \quad \text{and} \quad G_k = \begin{bmatrix} T^2 \\ T \cdot I_2 \end{bmatrix} \]

with \( T \) being the interval between the beginning of two consecutive observation periods, \( I_2 \) and \( 0_2 \) representing respectively the \( 2 \times 2 \) identity matrix and the \( 2 \times 2 \) null matrix.

2.2. The Observation Model

The considered multistatic radar system has \( M \) receivers and one transmitter, all equipped with one isotropic antenna. The complex baseband signal \( y_{k,m}(t) \) at the output of the \( m \)th receiver in the \( k \)th observation interval \( t \in (t_{k,m}, t_{k,m} + T_s) \), where \( t_{k,m} \) and \( T_s \) denote respectively the beginning and the duration of data acquisition (\( T_s \leq T \)), can be written as

\[ y_{k,m}(t) = x_{k,m}(t; \phi_k, \alpha_{k,m}) + w_{k,m}(t) \]

with \( x_{k,m}(t; \phi_k, \alpha_{k,m}) \) representing the target signal, and \( w_{k,m}(t) \) being a complex zero-mean circularly symmetric Gaussian noise process with spectral height \( \sigma_w^2 \). The target signal \( x_{k,m}(t; \phi_k, \alpha_{k,m}) \) is modelled as

\[ x_{k,m}(t; \phi_k, \alpha_{k,m}) = \alpha_{k,m} \cdot \exp\left\{j2\pi f_c \tau_{k,m}(t; \phi_k)\right\} \cdot u(t - \tau_{k,m}(t; \phi_k)), \]

where \( \phi_k = (\theta_k, n_k) \) denotes the motion-related parameters of the target, \( \alpha_{k,m} \) denotes the complex gain, \( f_c \) is the carrier frequency, and \( u(t) \) represents the transmitted signal. The delay \( \tau_{k,m}(t; \phi_k) \) of the target signal can be written as

\[ \tau_{k,m}(t; \phi_k) = \left( \|p_{tx} - r_k(t; \phi_k)\| + \|p_{rx,m} - r_k(t; \phi_k)\| \right) / c, \]

where the vectors \( p_{tx} \) and \( p_{rx,m} \) denote respectively the location of the transmitter and the location of the \( m \)th receiver, \( \| \cdot \| \) represents the Euclidean norm, \( c \) is the speed of light, and the target trajectory \( r_k(t; \phi_k) \) within the acquisition period \( t \in (t_{k,m}, t_{k,m} + T_s) \) is calculated as

\[ r_k(t; \phi_k) = p_k + (t - t_{k,m}) \cdot v_k + (t - t_{k,m})^2 \cdot n_k / 2. \]

In this contribution, we consider the scenario where the received signal \( y_{k,m}(t) \) in (3) is sampled at \( N \) discrete-time instants, i.e., \( t_1, t_2, \ldots, t_N \) with sampling rate \( T_s \). For notational convenience, the vector

\[ y_{k,m} = [y_{k,m}(t_1), \ldots, y_{k,m}(t_N)]^T \]

is used to denote all samples received at the \( m \)th receiver and the \( k \)th observation period, \( y_k = [y_{k,1}^T, y_{k,2}^T, \ldots, y_{k,M}^T]^T \) represents the samples obtained at all receivers in this period, and \( y_{1:k} = (y_1, y_2, \ldots, y_k) \) denotes all samples received up to the \( k \)th observation period.

3. THE PARTICLE FILTER

From (1) and (3), we see that the received signal \( y_k \) in a single-target scenario depends only on the current parameters \( \Omega_k = (\Phi_k, \alpha_k) \) with \( \alpha_k = [\alpha_{k,1}, \ldots, \alpha_{k,M}]^T \) and is conditionally independent of the other states given \( \Omega_k \). Utilizing this property, a sequential Monte-Carlo method [9] can be used to estimate the posterior probability density function (pdf) \( p(\Omega_{1:t} | y_{1:k}) \) where \( \Omega_{1:k} = (\Omega_1, \ldots, \Omega_k) \) denotes the sequence of the parameters of a target up to the \( k \)th observation period. In the following, we propose a PF to estimate \( \Omega_{1:k} \) sequentially from \( y_{1:k} \). Using this algorithm to track multiple targets is discussed at the end of this section.

3.1. Initialization

The parameters characterizing the \( i \)th particle in the \( k \)th observation reads \( \Omega_{k}^{i} = (\Phi_k, \alpha_k^i), \ i = 1, \ldots, I \), where \( I \) is the total number of particles. The parameters in the first observation, i.e., \( \Omega_1^i, \ i = 1, \ldots, I \), are initialized using the ML estimates of \( \Omega_1 \), i.e.,

\[ \Phi_1^i = \left( \hat{\phi}_1 \right)_{\text{ML}}, \]

\[ \alpha_1^i = \left( \hat{\alpha}_1 \right)_{\text{ML}}, \]

\[ \hat{\phi}_1 = \arg \max_{\phi_1} |y_{1}^u s(\phi_1)|, \quad \text{and} \quad \hat{\alpha}_1 = \left( \hat{\alpha}_1 \right)_{\text{ML}}, \]

\[ \hat{\alpha}_1 = \frac{s_{1,m}(\hat{\phi}_1_{\text{ML}})}{|s_{1,m}(\hat{\phi}_1_{\text{ML}})|^2}, \quad m = 1, \ldots, M \]
with $(·) H$ representing the Hermitian operation, the vector $s_{1,m}(φ_1) = [s_{1,m}(t_1; φ_1), \ldots, s_{1,m}(t_N; φ_1)]^T$ with
\[ s_{1,m}(t; φ_1) = \exp(j2πf_cτ_{1,m}(t; φ_1)) \]
and the vector $s(φ_1) = [s_{1,1}(φ_1), \ldots, s_{1,M}(φ_1)]^T$.

3.2. The Steps of the Particle Filter

When a new observation, say $y_k$, is available, the PF performs the following steps.

**Step 1: Predict the parameter vectors of particles and calculate the importance weights.**

The PF output from the previous observations is $(Ω_{k-1}^i, w_{k-1}^i), i = 1, \ldots, I$, where $w_{k-1}^i$ denotes the importance weight of the $i$th particle. We first predict the parameters of all particles for the $k$th observation. The values $n_k^i$, $i = 1, \ldots, I$ are drawn from $N(0, Σ_n)$, where the acceleration covariance matrix $Σ_n$ can be predetermined based on the kinematic feature and the maneuver hypothesis of the target.

The vectors $p_k^i$, $v_k^i$ and $α_k^i$ are calculated as respectively
\[ p_k^i = p_{k-1}^i + Tw_{k-1}^i + T^2n_{k-1}/2, \]
\[ v_k^i = v_{k-1}^i + Tn_{k-1}, \]
\[ α_{k,m}^i = \frac{s_{k,m}(φ_k^i)y_{k,m}}{||s_{k,m}(φ_k^i)||^2}, m = 1, \ldots, M. \]

The importance weights of the particles are computed as
\[ \tilde{w}_k^i = \frac{w_{k-1}^i p(y_k|Ω_k)}{\sum_{i=1}^I w_{k-1}^i p(y_k|Ω_k)}, \quad i = 1, \ldots, I, \]
where the posterior probability $p(y_k|Ω_k)$ is given by
\[ p(y_k|Ω_k) = (2πσ_0^2)^{-N} \cdot \exp\left(-\frac{1}{2σ_0^2}||y_k - x_k^i||^2\right) \]
with $x_k^i$ containing the entries $x_{k,m}(t; φ_k^i, α_{k,m}^i), t = t_1, \ldots, t_N$ and $m = 1, \ldots, M$.

**Step 2: Additional resampling.** In the considered radar system with multiple receivers and using wideband transmission, e.g. of DVB-T signals, for target illumination the amount of temporal-spatial samples available in one observation period is usually large. As a consequence, a significant mass of the posterior pdf $p(Ω_{k}|y_{1:k})$ is concentrated around its modes. As the target moves, the particles that are generated in **Step 1** can be too diffuse to “catch” the posterior probability mass. As a result, the importance weights of these particles are with high probability close to zero, even numerically uncomputable when using MATLAB. One brute-force solution is to employ a large number of particles. However, the resulting complexity prohibits any practical implementation. This problem can be overcome with low complexity by using the local-likelihood-sampling and local-importance-sampling methods [10]. Because both methods introduce window functions in the computation of the particle weights, the particles generated do not approximate the true posterior pdf $p(Ω_{1:k}|y_{1:k})$, and consequently, the estimation results can be artifacts.

In this contribution, we introduce an additional resampling step in which two techniques are used for allocation of particles without misinterpreting the posterior pdf. This step is activated when the importance weights of the particles obtained from (12) are so small that they are numerically uncomputable by using, e.g. MATLAB. The first technique consists in dropping some components in $y_k$, when calculating the importance weights. We denote the remaining components of $y_k$ by $\tilde{y}_k$. As the number of components in $y_k$ is less than that in $\tilde{y}_k$, the posterior pdf $p(Ω_k|y_{1:k})$ is less concentrated than the original pdf $p(Ω_k|y_{1:k})$. Thus, the particles have higher probability to get significant importance weights allocated.

The second method consists in computing the importance weights as
\[ \tilde{w}_k^i = \log(p(y_k|Ω_k^i) + \log w_{k-1}^i. \]

The obtained set $\{Ω_k^i, \tilde{w}_k^i\}$ is an estimate of the function $\log(p(Ω_{1:k}|y_{1:k}))$. This function exhibits the same modes as $p(Ω_{1:k}|y_{1:k})$ but it has a wider curvature in the vicinities of the modes. So, the probability to get non-negligible importance weights is enhanced.

Based on these two methods, we propose an additional resampling step, which can be implemented with the following pseudo-code.

```
for n = 1 to N do
  Step 2.1 Select vector $\tilde{y}_k^n$ which contains some of the components of $y_k$. The number of components in $\tilde{y}_k^n$ increases with respect to $n$.
  Step 2.2 Calculate the importance weights $\tilde{w}_k^n$, $i = 1, \ldots, I$ according to (14) with $y_k$ replaced by $\tilde{y}_k^n$.
  if $\{\tilde{w}_k^n\}$ contains non-significant values, e.g. less than $\max(\tilde{w}_k^n) - 3$, then
  Step 2.3 Find the indices $A = \{i^n\}$ of the particles with significant importance weights. Let $D$ denote the number of particles with non-significant weights.
  Step 2.4 Generate $D$ new particles using the parameter vectors $Ω_k^{(A_d)}$, $d = 1, \ldots, D$, according to **Step 1**. Here, $A_d$ denotes the $d$th element of $A$, and $j(A_d)$ is the index of a particle in the $(k-1)$th observation from which the $A_d$th particle in the $k$th observation is generated. Replace the particles that have non-significant weights by the new particles.
  Step 2.5 Update the importance weights $w_{k-1}^i$ as
\[ w_{k-1}^i = J(i)^{-1}w_{k-1}^i, \quad i = 1, \ldots, I, \]
where $J(i)$ represents the total number of new particles generated using the $j(i)$th particle in the $(k-1)$th observation. Go to **Step 2.2**.
end if
end for
```
**Step 3: Standard resampling.** The operations performed in this step are similar to those shown in the loop in Step 2 except that the importance weights $\hat{w}_k^i$ are replaced by $w_k^i$ and the observation $\tilde{y}_k^n$ is substituted with $y_k^n$.

**Step 4: Estimate the posterior pdf.** The estimate of the posterior pdf can be approximated with the particles and their importance weights according to

$$
\hat{p}(\Omega_{1:k}|y_{1:k}) = \sum_{i=1}^{I} w_k^i \delta(\Omega_{1:k} - \Omega_{1:k}^i).
$$

(16)

This pdf estimate can be used to estimate the expectation of a function of $\Omega_{1:k}$. For example, the state vector $\Omega_{1:k}$ for the target can be estimated as

$$
\tilde{\Omega}_{1:k} = \sum_{i=1}^{I} \Omega_{1:k}^i w_k^i.
$$

(17)

### 3.3. Extension to Multi-Target Scenarios

In a multi-target scenario where the targets are well separated, the proposed PF can be multiplicated to a bank of PFs, each one tracking an individual target. Targets are considered to be well-separated if the delay difference of the target signals from at least one of the receivers is larger than the intrinsic delay resolution of the radar system. The parameters of the particles in each PF can be initialized using the parameter estimates obtained with the Space-Alternating Generalized Expectation-maximization (SAGE) algorithm [11]. This algorithm calculates approximations of the ML parameter estimates in the multi-target scenarios with tractable computational complexity. To improve the tracking accuracy and robustness, successive interference cancellation methods [12] can be applied to mitigate the influences from the other targets when tracking a specific target. Due to the space limitation, the implementation details of the PF in multi-target scenarios is not described in this contribution.

### 4. SIMULATION STUDIES

The performance of the proposed algorithm is evaluated via simulations in a single-target scenario first and then in a two-target scenario. The considered multistatic passive radar system has three receivers and one transmitter. A DVB-T signal defined in [13] is used as “illuminator of opportunity”. The acquisition of the target signals is performed in a time-division multiplexing mode as depicted in Fig. 1. The data acquisition period $T_s$ is set equal to the duration of one DVB-T orthogonal frequency division multiplexing (OFDM) symbol, and the interval $T'$ between two consecutive observation periods equals 2000 · $T_s$.

Fig. 2 illustrates the geometrical constellation of the transmitter and the receivers considered in all simulations. The receivers are located close to each other within an area of $2 \times 2$ km² on a horizontal plane as depicted in Fig. 2, while the transmitter is 23 km away from the receiver closest to it.

An example of a target trajectory is shown in Fig. 2. In both scenarios the targets have the (same) speed equal to 200 m/s.

In the simulations, we assume that the locations of the transmitter and receivers are known, and that the transmitted signal $u(t)$ in (4) has been estimated perfectly using a reference signal received directly from the transmitter. We consider the case where the signals generated by other objects in the environment, such as clutter, are completely removed, so that the signals fed into the tracking algorithms are only the target signals and noise.

Fig. 3 (a) and Fig. 3 (b) depict the a-posterior pdf $p(\Omega_{k}|y_k)$ for two examples in a single-target scenario. The trajectory of the target is indicated as white curves in these figures. In both examples, $p(\Omega_{k}|y_k)$ exhibits a global maximum that is located at the true target position. This location coincides with the intersecting point of three ellipses. The foci of each ellipse are the location of the transmitter and the location of one of the receivers. This observation shows that the trajectory of a target, parameterized by $\Omega_k$, can be estimated by maximizing $p(\Omega_{k}|y_k)$ with respect to $\Omega_k$.

Fig. 4 depicts $p(\Omega_{k}|y_k)$ in the two-target scenario with the spacing $d$ of the targets as a parameter ranging from 5 m to 200 m. The signal-to-noise ratio (SNR) equals 50 dB. We observe that for $d < 40$ m, the a-posterior pdfs exhibit a single lobe. For $d \geq 40$ m, two lobes appear, with the locations of their maxima coincident with the true target positions. Calculation also shows that for $d > 40$ m, the delay difference of the target signals observed from at least one of the receivers is larger than the intrinsic range resolution of the considered radar system, i.e. 0.11 μs. In such a case, the target positions can be resolved by searching the local maxima of the posterior pdf $p(\Omega_{k}|y_k)$.
The proposed PF consumes around one tenth of the CPU time required when implementing the two conventional algorithms.

Fig. 5 depicts the estimated trajectories obtained by using the proposed PF and two conventional tracking algorithms in a single-target scenario. The two latter algorithms use a PF and an extended Kalman filtering (EKF) respectively to track the target trajectory from the estimates of Doppler frequency and delay of the target signal. These estimates are calculated by using a sample-based ML method applied to the samples of the target signals received from individual observation intervals. The standard deviations of the delay and Doppler frequency estimates are 0.275 samples (≈ 0.3 ns) and 54 Hz respectively. The SNR equals 30 dB. We observe from Fig. 5 that the trajectory estimated with the proposed PF algorithm fits the true trajectory well, while the trajectory estimates obtained with the conventional algorithms exhibit significant deviations. We calculate the root mean-square estimation error of the target position as

$$RMSEE(r_{1,K}) = \left( \frac{1}{KT} \sum_{k=2}^{K} \int_{0}^{T} \left\| r_k(t; \hat{\phi}_k) - r_k(t; \phi_k) \right\|^2 dt \right)^{1/2}$$

with $\hat{\phi}_k$ denoting the estimates of $\phi_k$. For the proposed PF, the conventional EKF-based algorithm and the PF-based algorithm, RMSEE($r_{1,K}$) equals 0.133 m, 90.8 m and 134.8 m respectively.

Fig. 6 depicts RMSEE($r_{1,K}$) versus the SNR obtained for the three considered tracking algorithms in a single-target scenario. The proposed PF returns RMSEE($r_{1,K}$) one order of magnitude less than those achieved with the two conventional methods. Using a larger number of particles, the proposed PF algorithm exhibits lower RMSE. When 600 particles are used, RMSEE($r_{1,K}$) obtained with the new PF decreases linearly from 20 m to 10 cm for SNR within the interval $[-10, 30]$ dB. Simulations also show that the CPU time of the proposed PF increases linearly with respect to the number of particles. With 50 particles, the proposed PF consumes around one tenth of the CPU time required when implementing the two conventional algorithms.
Fig. 7. Estimated target trajectories obtained by using two proposed PFs in a two-target scenario, with SNR 0 dB.

Fig. 7 depicts the target trajectories estimated by using two separate PFs, each equipped with 50 particles, in a two-target scenario. The true trajectories are separated by 65 m in the first 20 observation periods. The SNR equals 0 dB. It can be observed from Fig. 7 that the PF is capable of tracking the target individually, and the estimated trajectories fit well with the true trajectories. The RMSEE of target position equals 31.4 meters and 10.9 meters for the first and second target respectively.

5. CONCLUSIONS

We proposed a single-stage tracking algorithm based on a particle filter (PF) to track targets in a passive radar system. This algorithm estimates directly the trajectory of the target position in a Cartesian coordinate system from the received complex baseband signal, without performing any intermediate stages that are necessary in the conventional tracking algorithms. We implemented this method in a multistatic radar system with three receivers and one transmitter. A Digital-Video-Broadcasting Terrestrial (DVB-T) signal is used as the “illuminator of opportunity”. Numerical results show that the PF outperforms two conventional algorithms implementing multiple successive stages, in terms of tracking accuracy, robustness and computational complexity.

It is worth mentioning that the state-space model presented in this contribution is applicable for characterizing the motion of a mobile terminal or an object in a wireless network. The observation model can also be generalized to describe the complex band signals originating from the mobile terminal or from the interactions with the object. The proposed PF-based tracking algorithm can be extended for localization and tracking of mobile terminals or moving objects in e.g. wireless communication networks and wireless sensor networks.

6. REFERENCES


